Particle Swarm Optimization with Pade Approximation Based-Model Reduction of Automatic Voltage Regulator

*Nasir Ahmad Alawad, **Nora Ghani Rahman

**Assistant Professor, **Msc. student*

Department of Computer Engineering, Almustansiriyah University, IRAQ

Abstract - *An important entity for the operation of the energy scheme is the automatic voltage regulator (AVR). Therefore, studying its modeling is essential to observe the conduct and efficiency of AVR. Because AVR is a complicated scheme with a greater feature of order transfer, studying it is tedious. In this document, we have applied and implemented various classical model order reduction systems and blended Particle Swarm Optimization with Pade Approximation (PSO+Pade) compared to the suggested technique. MATLAB simulation was used to demonstrate the efficacy of the suggested technique relative to others.*

Keywords - *AVR, Model order reduction Techniques, PSO.*

I. INTRODUCTION

Physical systems such as aircraft, chemical plants, electrical power system networks, urban traffic networks, digital communication networks, financial systems and control system can be mathematically depicted by state space models or transfer function models in current-day technology. High-order model of the scheme is acquired from theoretical factors in the most practical control system circumstances. The higher-order model has many analytical and design problems [1]. For instance, the analysis and design requires more computational time, big complex hardware requirements, etc. Therefore, a decreased model that maintains the significant characteristics of a greater order scheme is desirable [2].Linear system approximation plays an significant role in many engineering issues, particularly in the design of control systems. Main goals of reducing model order are: [3]

1-To reduce the computational complexity of the model for analysis of system.
2-To simplify the understanding of system for design purpose.
3-To economics in terms of hardware when realizing the system.
4-To reduce the simulation time with the model of system.
5-To reduce the controller size.

The issue of model order decrease has been widely studied in literature[4-16], several techniques for large-scale system modelling are accessible[17]. Most of the standard techniques that have been established to date are accessible in a constant domain[18]. However, in both a continuous and a discrete domain, the high-order systems can be reduced.

Evolutionary Techniques [19-20], an area that uses analogies with nature or social systems, has been one of the most promising study areas in latest years. These methods are biologically based stochastic search techniques. Particle Swarm Optimization (PSO) and Genetic Algorithms (GAs) have recently emerged as a promising algorithm for dealing with optimization issues [21-22]. Unlike GAs, though, PSO has no evolution operators, such as crossover and mutation. One of the most encouraging points of interest of PSO over the GAs is its algorithmic effortlessness when it utilizes a couple of parameters and simple to execute.

In this article, the (PSO) technique is used to find the decreased system denominator because it is very efficient to give system poles values (this is very essential to assign system dynamics) and mixed with Differentiation te chnique to give the reduced system numerator(zeros), more information can be seen in the subsequent parts.

The paper is organized as follows. Section two overview the (AVR), section three devoted to different classical methods for comparison with proposed method, (DPSO) is explained in detail in section four and finally the discussion and conclusion appeared in setion5 and six respectively.

II. AVR MODELLING

(AVR) is the main controller in the excitation scheme that retains a synchronous generator terminal voltage at a given rate. Different kinds of excitation systems exist, depending on the technique of providing DC power[23]. We linearzed the AVR plant and linearzed parts to study this scheme[23,24].

These subsystems ' descriptions are as follows.

A- Amplifier Model

In excitation system, using an electronic device called amplifier, the amplitude of a signal waveform can be increased. The amplifier is represented by a gain Ka and a time constant Ta and the transfer function is[25, 26];-

$$G_a(s) = \frac{K_a}{1+\tau_a s} \quad (1)$$

Where K_a = [10, 400], τ_a = [0.02, 0.1] sec

B- Exciter Model

There is a variety of different excitation types. Though, contemporary excitation scheme use ac power source from beginning to end solid-state rectifiers such as SCR. In the simplest form, the transfer function of a modern exciter may be represented by a single time constant τ_e and a gain K_e, i.e. [25,26]

$$G_e(s) = \frac{K_e}{1+\tau_e s} \quad (2)$$

Where K_e= [0.8, 1], τ_e = [0.5, 1.0] sec.

C- Generator Model

For AVR, we have considered a simplified linearized model as shown in Figure. 1. The transfer function of generator model can be expressed as[23,25,26]

$$G_{OL}(s) = \frac{2HK_3K_6S^2+K_3K_6K_DS+K_3\omega_0(K_1K_6-K_2K_5)}{2HK_3S^3+(2H+K_DT_3)S^2+(K_D+\omega_0T_3)S+\omega_0(K_1-K_2K_3K_4)} \quad (3)$$

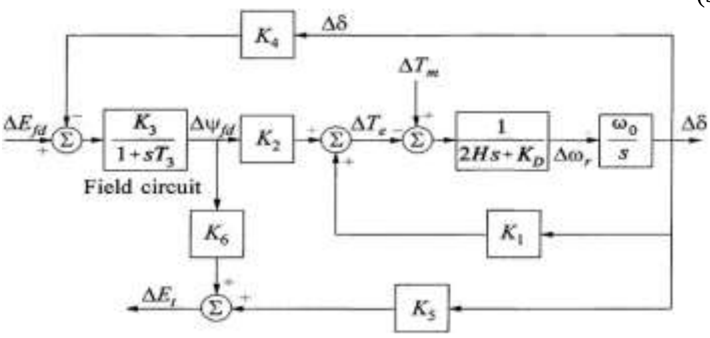

Fig.1. The generator model

D- Sensor Model

The voltage is identified by a prospective transformer and is modified by an extension rectifier. The sensor is modelled by a simple first order transfer function, given by a gain $K_s$and a time constant $\tau_s$will be[25,26] :-

$$H_s(s) = \frac{K_s}{1+\tau_s s} \quad (4)$$

Where Ks=[0.1 1] and τ_s = [0.01 0.7].

The open-loop transfer function of AVR ,can be represented as [25,26]:-

$$G_{AVR} = G_a * G_e * G_{ol} \quad (5)$$

Or can be written as:-

$$G_{AVR} = \frac{a_2s^2+a_1s+a_0}{b_5s^5+b_4s^4+b_3s^3+b_2s^2+b_1s+b_0} \quad (6)$$

Where

$$a_0 = K_aK_3\omega_0(K_1K_6 - K_2K_5)$$
$$a_1 = K_aK_eK_6K_D$$
$$b_5 = 2H\tau_AT_3\tau_e$$

$$b_4 = 2HT_3\tau_e + 2HT_3\tau_a + 2H\tau_e\tau_a + K_DT_3\tau_e\tau_a$$
$$b_3 = 2HT_3 + 2H\tau_e + K_DT_3\tau_e + 2H\tau_a + K_DT_3\tau_a + K_D\tau_e\tau_a$$
$$b_2 = 2H + K_DT_3 + K_D\tau_e + \omega_0T_3\tau_e + K_D\tau_a + \omega_0T_3\tau_a + \omega_0\tau_eK_1\tau_a - \omega_0\tau_eK_2K_3K_4\tau_a$$
$$b_1 = K_D + \omega_0T_3 + \omega_0K_1\tau_e - \omega_0\tau_eK_2K_3K_4 + \omega_0\tau_eK_1 + \omega_0\tau_aK_2K_3K_4$$
$$b_0 = \omega_0(K_1 - K_2K_3K_4)$$

Refer to ref. [ ], Where
K_1=1.591, K_2=1.5, K_3=0.333, K_4=1.8, K_5=0.12, K_6 =0.3,
T_3 =1.91, H =3, K_D=0, ω_0=377.
The open-loop transfer function of AVR as:

$$G_{AVR} = \frac{5.994s^2+825.2}{0.573s^5+7.176s^4+51.06s^3+451.1s^2+876.6s+260.8} \quad (7)$$

Equation (7) shows the open-loop transfer function of AVR, it is necessary to dealing with closed-loop system to measure the dynamic transient response parameters (max overshot, settling and rise times, etc), so that it must be insert the feedback measurement sensor.

Figure (2) shows the arrangement of the AVR system parts as a closed loop. The generator's terminal voltage Vt(s) is constantly detected by the sensor and compared to the required reference voltage Vref(s). The distinction between the reference and the sensed terminal voltages (e.g. error voltage) is amplified by the amplifier and used by the exciter to excite the generator.

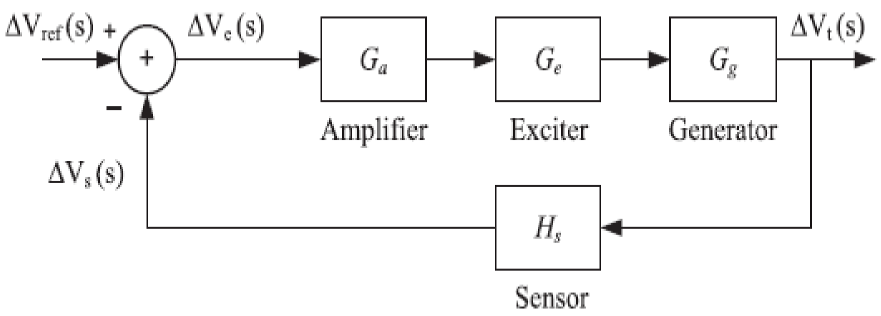

Fig.2. Simplified AVR block diagram

Finally, the closed-loop transfer function (after connecting the sensor feedback element) is:-

$$\frac{V_t(s)}{V_{ref}(s)} = \frac{G_{AVR}}{1+G_{AVR}H_S} \quad (8)$$

$$\frac{V_t(s)}{V_{ref}(s)} = \frac{0.0599s^3+5.994s^2+8.252s+825.2}{0.00573s^6+0.644s^5+7.686s^4+55.571s^3+465.86s^2+879.208s+1086} \quad (9)$$

III. CLASSICAL MODEL ORDER REDUCTION(CMOR)

To achieve the streamlined AVR model outlined in Eq (9), we apply various MOR algorithms. The brief description and decreased model acquired from the systems implemented are listed below; however, owing to page restriction, the comprehensive algorithm and formulation are not provided in the document. Note that we have attempted to approximate the sixth-order AVR plant to second in all the decrease systems implemented.

A- Balanced realization(BR)

The idea of balanced realization of a stable transfer function is extracted in statistical methods from the notion of main component assessment: i.e., controllability and observability grammars should be decomposed into main parts for assessing the contribution of each mode [27]. The decreased second-order model of the AVR scheme acquired using this method is:

$$R(s) = \frac{0.2638\,s + 0.6962}{s^2 + 0.7116\,s + 1.213} \qquad (10)$$

Figure 3 shows the transient response between original and reduced model using(BR)

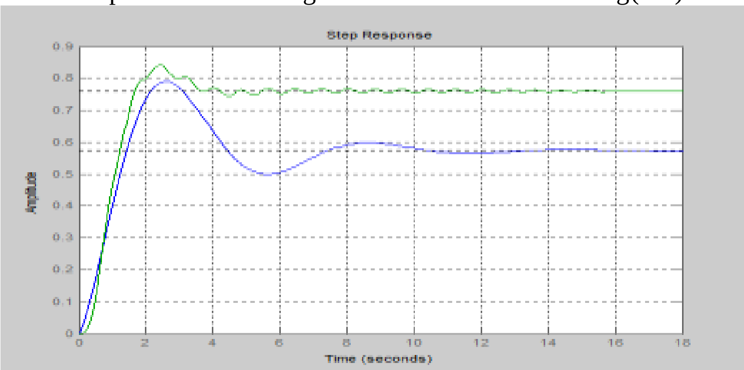

Fig.3. Step response of reduced order model AVR system using (BR)
Green line(original system), blue line(reduced system)

B- Time moment matching(TMM)

This method basically matches the set of time functions for original model with reduced order model without having to obtain the original model's time or frequency responses. From this scheme, we get the reduced order model of AVR system as[28]

$$R(s) = \frac{0.75985 - 0.0653s}{1 + 0.3326s + 0.0031s^2} \qquad (11)$$

Figure 4 shows the transient response between original and reduced model using(TMM)

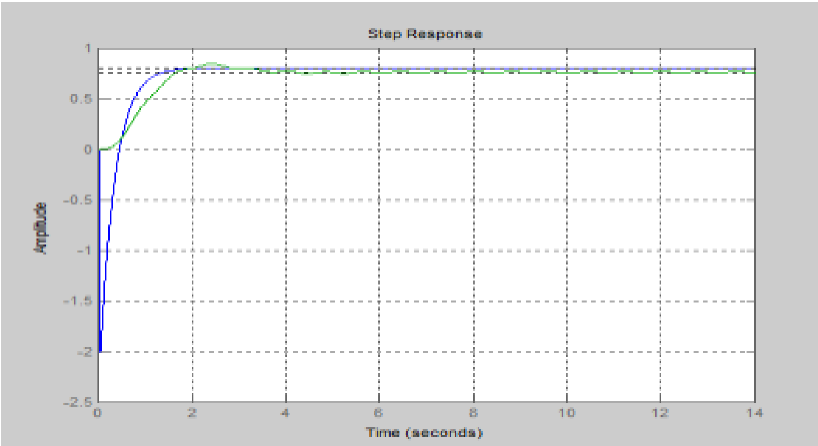

Fig.4. Step response of reduced order model AVR system using (TMM)

C- Truncation method(TM)

This is a simple and computationally efficient method of reduction which is based on simply truncating the high order terms in system transfer function to get the reduced order model. Using this technique [28], the reduced second-order model of AVR system obtained is:

$$R(s) = \frac{8.252s + 825.2}{465.86s^2 + 879.208s + 1086} \qquad (12)$$

Figure 5 shows the transient response between original and reduced model using Truncation method.

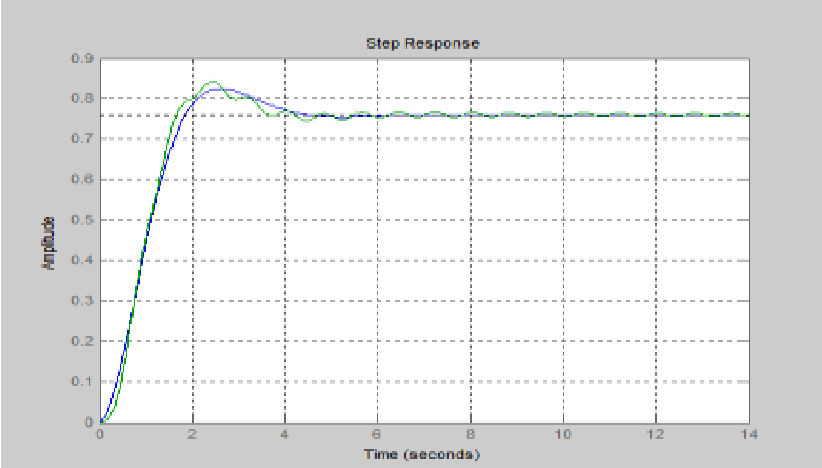

Fig.5. Step response of reduced order model AVR system using(TM)

D- Routh Hurwitz stability criteria(RHS)

This method is based on formation of Routh array expansion, numerator stability routh array and denominator stability routh array is formed and appropriate coefficients for required reduced order is chosen to get a reduced order model[29]. Using this method, we get the model as:-

$$R(s) = \frac{0.0055s+825.2}{71.6445s^2+561.3489s+1086} \quad (13)$$

Figure 6 shows the transient response between original and reduced model using(RHS)

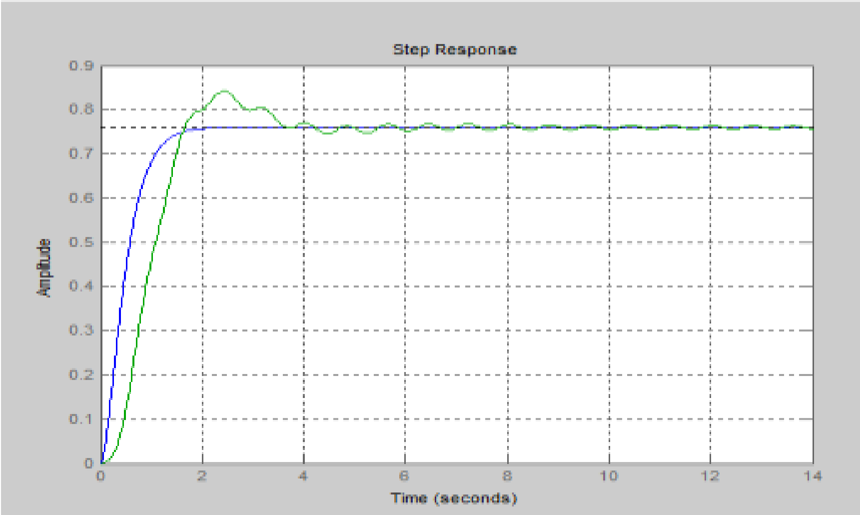

Fig.6. Step response of reduced order model AVR system using RHS

E- Routh alpha beta approximation(RAB)

In this method denominator and numerator of system transfer function are brought into Routh form and corresponding α 's and β ' s are obtained respectively, and the reduced order model can be found[30],Thus, using this reduction scheme, we get the second-order reduced model as:-

$$R(s) = \frac{0.0055s+825.2}{71.6445s^2+561.3489s+1086} \quad (14)$$

Figure 7 shows the transient response between original and reduced model using(RAB)

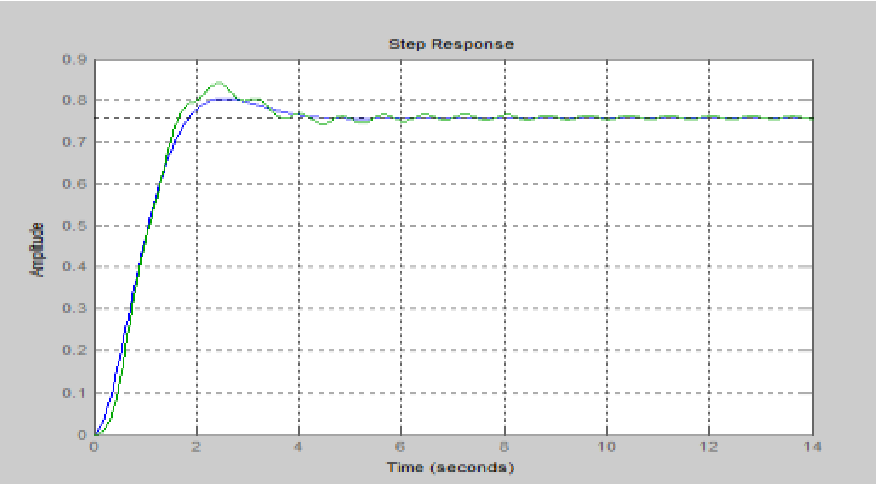

Fig.7. Step response of reduced order model AVR system using RAB

F- Stability equation method(SEM)

In this technique the transfer function of reduced orders are obtained directly from the pole zero patterns of the stability equations of the original transfer function [31]. Therefore the order of the transfer functions stability equations can be decreased.

$$\mathbf{R(s)} = \frac{N_r(s)}{D_r(s)} = \frac{8.252s+825.2}{465.86s^2+879.208s+1086} \qquad (15)$$

Figure 8 shows the transient response between original and reduced model using(SEM)

Fig. 8. Step response of reduced order model AVR system using SEM

IV. Proposed Method(PADE+PSO)

The proposed method is mixing between Pade approximation method and optimization technique (PSO).

A- Pade approximation method

This approach stems from the theory of Pade and was later used for model reduction by Shamash [33]. Consider a function is:-

$f(s) = c0 + c1s + c2s2 + \ldots\ldots\ldots$ (16)

For the function *f*(*s*) in Eqn. (16) to be approximated, let the following Pade approximant be defined.

$$\frac{U_n(s)}{V_n(s)} = \frac{a_0 + a_1 s + a_2 s^2 + \cdots + a_{n-1} s^{n-1}}{b_0 + b_1 s + b_2 s^2 + \cdots + b_{n-1} s^{n-1} + s^n} \tag{17}$$

For the first (*m* + *n*) terms of Eqn. (16) and Eqn. (17) to be equivalent, it becomes apparent that the following set of relations must hold:

a0 = b0c0
a1 = b0c1 + b1c0 (18)
a2 = b0c2 + b1c1 + b2c0
...
an-1 = b0cn-1 + b1cn-2 + ….. + bn-1c0
0 = b0cn + b1cn-1 + … + bnc0
...
0 = b0c2n-1 + b1c2n-2 + ….. + bn-2cn + cn-1
Once the coefficients *c i* = 0*;* 1*;* 2*;* ... are found out using Eqn:-

$$a_{k,l} = a_{k-1,1}.a_{1,l+1} - a_{k-1,l+1}.a_{1,1} \tag{19}$$

and *cj* = (¡1)*jaj*+2;1,
Eqns. (18 and19) can be written in vector form as:-

$$\begin{bmatrix} c_n & c_{n-1} & \cdots & & c_1 \\ c_{n+1} & c_n & c_{n-1} & \cdots & c_2 \\ c_{n+2} & c_{n+1} & c_n & \cdots & c_3 \\ \vdots & & & \ddots & \\ & & & & c_n \\ c_{2n-1} & c_{2n-2} & \cdots & & c_n \end{bmatrix} \begin{bmatrix} b_0 \\ b_1 \\ b_2 \\ \vdots \\ b_{n-2} \\ b_{n-1} \end{bmatrix} = \begin{bmatrix} -c_0 \\ -c_1 \\ -c_2 \\ \vdots \\ -c_{n-2} \\ -c_{n-1} \end{bmatrix}$$

$$\begin{bmatrix} c_0 & 0 & \cdots & & & 0 \\ c_1 & c_0 & 0 & \cdots & & 0 \\ c_2 & c_1 & c_0 & 0 & \cdots & 0 \\ \vdots & \vdots & & \ddots & 0 & \vdots \\ & & & & & 0 \\ c_{n-1} & c_{n-2} & \cdots & & c_1 & c_0 \end{bmatrix} \begin{bmatrix} b_0 \\ b_1 \\ b_2 \\ \vdots \\ b_{n-2} \\ b_{n-1} \end{bmatrix} = \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ \vdots \\ a_{n-2} \\ a_{n-1} \end{bmatrix}$$

In this work, needs only the denominator of Eq(17),for full model order Eq(9),we get after using above equations ,the denominator of reduced order is:-

$$D(s) = s^2 + 2.095s + 2.993 \tag{20}$$

B- PSO method

PSO is a population-based technique, that is, by using a population that is iteratively altered until a termination criterion is met, it reflects the algorithm country. PSO algorithms frequently refer to the population

P={p1,...,pn} of the possible alternatives as swarm. Particles are the viable alternatives p1,...,pn. The PSO technique views the Rd set of options as a "space" in which the particles "move." The range of particles is generally selected between 10 and 50 to solve practical issues. The ordinary goal of the PSO algorithm is to fix an unconstrained issue of minimization. PSO might sound complex, but it's a very simple algorithm. A group of variables have their values adjusted closer to the member whose value at any given moment is closest to the goal over a range of iterations. There are generally many papers on the PSO algorithm and how to solve the problems.

The integral square error criteria(ISE) is used in this document as a performance index to reduce the mistake bet ween initial and decreased system parameters. The PSO algorithm parameters are regarded as follows in this job:
1- Inertial weight: 0.9
2- Acceleration factors (c1 and c2): are (0.12)and (1.2)
3- Population size: 50
4- Maximum iteration: 200
The objective function (OF) must be a minimization problem. Let us suppose that there is a problem defined is needed to be optimized by using:-

$$\text{ISE} = \int_0^{\infty} [e(t)]^2 \, dt, \tag{21}$$

After using PSO MATLAB program, it is found that the numerator of reduced order is:-

$$N(s) = -0.4063s + 2.278 \tag{22}$$

So that the reduced order transfer function from Eq(20) andEq(22) is:-

$$\frac{N(s)}{D(s)} = \frac{-0.4063\,s + 2.278}{s^2 + 2.095\,s + 2.993} \tag{23}$$

Figure 9 shows step response of reduced order model AVR system using mixed(PSO+Pade) method, where (go) is the original system and(gr)is reduced model.

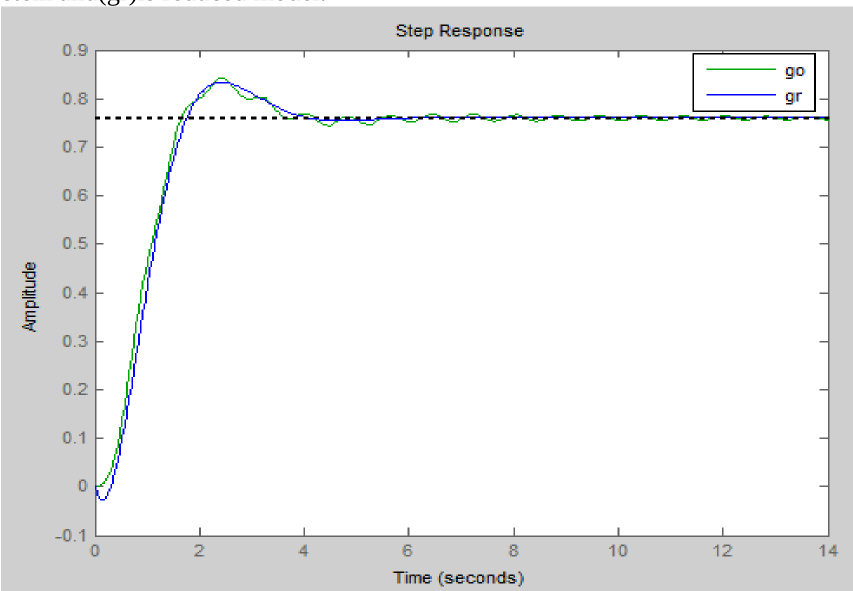

Fig.9. Step response of reduced order model AVR system using(PSO+Pade)

V. Discussion

A mixed method is suggested to use the pade approximation technique and PSO algorithm to determine the stable decreased order models. If the initial scheme is stable, this technique ensures decreased model stability.
This article provides a comparison of reduction methods for model order. This comparison gives us a measure of the decreased system's precision. By comparing this, we get to understand the best technique that can be used

to reduce model order. This article also provides a comparison for decreased system stability, as well as transient response parameters such as (maximum overshot, increase, settling, peak time, ISE performance index). Recently, the method of particle swarm optimization (PSO) has drawn significant attention among multiple contemporary methods of heuristic optimization.
The PSO method is based on minimizing the Integral Squared Error (ISE) between the transient responses of the original higher order model and the reduced order model of the unit step input, as shown in table(1), the objective function value ISE=0.0075, which is the lower value compared to other techniques.
Comparison of transient parameters between all decreased techniques is shown in Table 1. It is apparent that the mixed technique (PSO+Pade) provides the same transient parameters as the original system.

TABLE. 1 . Shows the performance comparison between all methods

Property	**original**	**RHS**	**TMM**	**RAB**	**SEM**	**BR**	**TM**	**Pso+Pade**
Rise time	1.03	0.871	0.71	1.23	1.24	1.08	1.24	1.06
Settling time	4.5	1.52	1.31	3.63	3.9	9.78	3.9	3.52
Overshoot	10.9	0	$2.22*\mathbf{10^{-14}}$	5.9	8.46	37.7	8.46	9.3
Peak	0.843	0.76	-2	0.805	0.824	0.79	0.824	0.831
Peak time	2.42	3	0.0318	2.54	2.59	2.59	2.59	2.37
Zeros	1.0e+02 * -1.0007 + 0.0000i -0.0000 + 0.1173i -0.0000 - 0.1173i	-15004	1.0823	- 100.0241	-100	-2.6392	-100	5.6066
Poles	-99.8497 + 0.0000i -10.2222 + 0.0000i -0.1133 + 7.8699i -0.1133 - 7.8699i -1.0462 + 1.3794i -1.0462 - 1.3794i	- 4.3528 - 3.4824	-104.1944 -3.0960	-1.1067 + 1.2285i -1.1067 - 1.2285i	-0.9436 + 1.2003i -0.9436 - 1.2003i	-0.3558 + 1.0425i -0.3558 - 1.0425i	-0.9436 + 1.2003i -0.9436 - 1.2003i	-1.0475 + 1.3769i -1.0475 - 1.3769i
ISE		0.7867	4.4387	0.0330	0.0258	13.0319	0.0258	0.0057

VI. CONCLUSION

PSO is a comparatively latest heuristic search method whose mechanics are influenced by biological populations ' swarming or cooperative behavior. The primary drawback of the suggested model order reduction technique provides the system a non-minimum phase (have one zero in the right half of the s-plane).
The response to a step input from a non-minimum phase system has an "undershoot," as shown in fig.9. The response to the phase was first negative, then positive. This may take more control operation if this scheme is applied in the feedback control loop.
In the Pade-PSO technique, there are no large differences between original and decreased order schemes, where both have almost the same description.
To demonstrate their superiority, the results collected are compared to a newly published classic technique and other current well-known techniques of reducing model order. Compared to any other order reduction technique, it is evident from the outcomes described in Table 1 that the suggested method gives minimum ISE error.

It can be seen that the steady state responses of the suggested decreased order models match the original model precisely.

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